



ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

**MCS 102 - Calculus for International Trade II**

## FINAL EXAMINATION

**STUDENT NUMBER:**

**NAME-SURNAME:**

**SIGNATURE:**

**INSTRUCTOR:**

**DURATION:** 80 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

### IMPORTANT NOTES:

- 1) Please make sure that you have written your student number and name above.
- 2) Check that the exam paper contains 5 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.

1) Solve the following integral questions.

a. Find the area between the curves  $y = 2x^2 - 20$  and  $y = -x^2 + 28$ .

b.  $\int \frac{\ln(2x - 4)}{x - 2} dx$

c.  $\int_{-2}^{-1} \frac{x + 5}{x + 3} dx.$

2) Let

$$A = \begin{pmatrix} 4 & 3 \\ 2 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix}.$$

a. Evaluate  $3C(A + 2B)^T$ .

b. If  $D$  is a  $3 \times 3$  matrix and  $E$  is a  $3 \times 3$  matrix where  $\det(D^{-1}) = 3$  and  $\det(E) = 6$ , then calculate

$$\det((DE)^{-1}), \quad \det(2D^{-1}).$$

State clearly your answers.

**3) a.** Let  $z^2 + x^2 + y^2 = 6$ . Use implicit partial differentiation to find  $z_x$  and  $z_y$ . Evaluate  $z_x$  and  $z_y$  for  $x = -1$ ,  $y = 1$  and  $z = 2$ .

**b.** Let  $f(x, y) = 2x^2 + 3y^2$  and  $x = s - 3r$ ,  $y = 6r + s$ . Then find  $z_r$  and  $z_s$  by means of chain rule and evaluate them at  $r = 0$  and  $s = 1$ .

4) Find the critical points of the function

$$f(x, y) = 3x^3 + y^3 - 9x - 12y + 5$$

and classify them by using the second derivative test.

5) Use Lagrange multipliers to find the extreme values of the function

$$f(x, y, z) = 3x + 2y + z$$

subject to constraint  $3x^2 + 2y^2 + z^2 = 24$ .