ÇANKAYA UNIVERSITY

Department of Mathematics

MCS 102 - Mathematics for International Trade II

SECOND MIDTERM EXAMINATION
24.04.2017

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

Question	Grade	Out of
1		30
2		25
3		30
4		20
Total		105

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 4 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$.

(a) Find A^{-1} by using elementary row operations.

(b) Find $\mathrm{adj}(A)$.

(c) Solve the system $Ax = b$ if $b = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$.

Solution:

(a) $A^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

(b) $\mathrm{adj}(A) = \det(A)A^{-1}$ and $\det(A) = -1 \Longrightarrow \mathrm{adj}(A) = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

(c) $x = A^{-1}b = \begin{bmatrix} 3 \\ -1 \\ -6 \end{bmatrix}$

2) Let $A = \begin{bmatrix} 2 & -3 & 0 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -6 \\ 5 & 2 & 3 \\ 4 & 1 & -6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 3 & 2 & 0 & 1 \\ 1 & -1 & 1 & 2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$.

Compute the following determinants:

(a) $\det(A) = 2$
(b) $\det(B) = 3$
(c) $\det(C) = -8$
(d) $\det(4B) = 4^3 \det(B) = 4^3 3 = 192$
(e) $\det(A^{-1}) = \dfrac{1}{\det(A)} = \dfrac{1}{2}$
(f) $\det(A^2 B^3) = \det(A)^2 \det(B)^3 = 2^2 3^3 = 108$
(g) $\det(A^T B) = \det(A) \det(B) = 6$

3) a) Solve the following linear system by using matrix reduction.

$$\begin{aligned} x + 2y &= 1 \\ x + 3y + 3z &= 2 \\ 2x + 3y - 3z &= 1 \end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 3 & -3 & 1 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -6 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad x = 6z - 1,\ y = -3z + 1$$

$z = t,\ x = 6t - 1,\ y = -3t + 1$

b) Solve the following linear system by using matrix reduction.

$$\begin{aligned} x + y + z &= 1 \\ x + y - 2z &= 3 \\ 2x + y + z &= 2 \end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right] \quad x = 1,\ y = 2/3,\ z = -2/3$$

c) Solve the following linear system by Cramer's rule.

$$\begin{aligned} x + 3y + 2z &= 2 \\ 2x - y + z &= 0 \\ 5x + 3z &= -1. \end{aligned}$$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ 5 & 0 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 2 & 3 & 2 \\ 0 & -1 & 1 \\ -1 & 0 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 5 & -1 & 3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 0 \\ 5 & 0 & -1 \end{bmatrix}$$

$\det(A) = 4$, $\det(A_1) = -11$, $\det(A_2) = -5$, $\det(A_3) = 17$

$$x_1 = \frac{|A_1|}{|A|} = \frac{-11}{4},\ x_2 = \frac{|A_2|}{|A|} = \frac{-5}{4},\ x_3 = \frac{|A_3|}{|A|} = \frac{17}{4}$$

4) Consider the matrices

$$A = \begin{bmatrix} 0 & 3 \\ -4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -4 & 1 \\ 1 & -4 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 7 \\ 0 & -2 \\ 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 6 \\ 0 \\ -3 \end{bmatrix}.$$

a) Calculate A^2.

$$A^2 = \begin{bmatrix} -12 & 3 \\ -4 & -11 \end{bmatrix}$$

b) Calculate B^T.

$$B^T = \begin{bmatrix} 3 & 1 \\ -4 & -4 \\ 1 & 7 \end{bmatrix}$$

c) Calculate BA if it is possible. If not explain why?

It is not possible since the number of columns of B not equal to the number of rows of A.

d) Calculate $BC - A^T$ if it is possible. If not explain why?

$$BC - A^T = \begin{bmatrix} 18 & 34 \\ 23 & 21 \end{bmatrix}$$

e) Calculate BD if it is possible. If not explain why?

$$BD = \begin{bmatrix} 15 \\ -15 \end{bmatrix}$$