ÇANKAYA UNIVERSITY

Department of Mathematics

MCS 102 - Mathematics for International Trade II

FIRST MIDTERM EXAMINATION
20.03.2017

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 70 minutes

Question	Grade	Out of
1		24
2		18
3		23
4		18
5		20
Total		103

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Evaluate.

a) (6 pnt) $\displaystyle\int \left(3x^5 - x\sqrt{x} + \frac{1}{x^3} - 5\right) dx = \frac{3x^6}{6} - \frac{2x^{5/2}}{5} + \frac{x^{-2}}{-2} - 5x + C$

b) (6 pnt)

$$\begin{aligned}\int_1^4 \frac{3x^4 - \sqrt{x} - 1}{x} dx &= \int_1^4 \left(3x^3 - x^{-1/2} - \frac{1}{x}\right) dx \\ &= \left(3\frac{x^4}{4} - 2x^{1/2} - \ln|x|\right)\Big|_1^4 \\ &= 190 - \ln 4 - \frac{3}{4}\end{aligned}$$

c) (6 pnt) $\displaystyle\int (x^3 - 1)\left(1 + \frac{1}{x}\right) dx = \int \left(x^3 + x^2 - 1 - \frac{1}{x}\right) dx = \frac{x^4}{4} + \frac{x^3}{3} - x - \ln|x| + C$

d) (6 pnt) $\displaystyle\int \frac{8x^3 - 8x + 2}{x^4 - 2x^2 + x} dx = \int \frac{2du}{u} = 2\ln|u| + C = 2\ln|x^4 - 2x^2 + x| + C$

$$u = x^4 - 2x^2 + x, \quad du = (8x^3 - 8x + 2)dx$$

2) Evaluate.

a) (6 pnt) $\int e^{x^3-x}(3x^2-1)dx = \int e^u du = e^u + C = e^{x^3-x} + C$

$$u = x^3 - x, \quad du = (3x^2-1)dx$$

b) (6 pnt) $\int (3x+1)\sqrt[3]{3x^2+2x}\,dx = \int \sqrt[3]{u}\frac{du}{2} = \frac{u^{4/3}}{4/3}\frac{1}{2} + C = \frac{3}{2}(3x^2+2x)^{4/3} + C$

$$u = 3x^2 + 2x, \quad du = (6x+2)dx$$

c) (6 pnt) $\int (1-3x^2)(x-x^3)^7\,dx = \int u^7 du = \frac{u^8}{8} + C = \frac{(x-x^3)^8}{8} + C$

$$u = x - x^3, \quad du = (1-3x^2)dx$$

3) a) (8 pnt) Evaluate $\int \frac{5x}{x^2-x-6}dx$

$$I = \int \frac{5x}{x^2-x-6}dx = \int \frac{5x}{(x-3)(x+2)}dx$$

$$\frac{5x}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \Longrightarrow A = 3, \quad B = 2$$

$$I = \int \left(\frac{3}{x-3} + \frac{2}{x+2}\right) = 3\ln|x-3| + 2\ln|x+2| + C$$

b) (7 pnt) Evaluate $I = \int x^5 \ln x dx$

$$u = \ln x, \ du = \frac{1}{x}dx, \ dv = x^5 dx, \quad v = \frac{x^6}{6}$$

$$\begin{aligned}\int udv =& uv - \int vdu \\ I =& \frac{x^6}{6}\ln x - \int \frac{x^6}{6}\frac{1}{x}dx \\ =& \frac{x^6}{6}\ln x - \frac{x^6}{36} + C\end{aligned}$$

c) (8 pnt) Evaluate $\int \frac{(e^{-x}+3)^5}{e^x}dx = \int u^5(-du) = -\frac{u^6}{6} + C = -\frac{(e^{-x}+3)^6}{6} + C$

$$u = e^{-x} + 3, \ \ du = -e^{-x}dx$$

4) a) (8 pnt) If y is a function of x such that $\dfrac{dy}{dx} = x - \dfrac{3}{\sqrt{x}}$ and $y(1) = -3$, find $y(x)$.

Solution:

$$\int dy = \int \left(x - \frac{3}{\sqrt{x}}\right) dx$$

$$y = \frac{x^2}{2} - 6x^{1/2} + C$$

$$y(1) = -3 \Longrightarrow C = \frac{5}{2}$$

$$y = \frac{x^2}{2} - 6x^{1/2} + \frac{5}{2}$$

b) (10 pnt) Find the area of the region between the curves $y = 9 - x^2$ and $y = 3 - x$.

Solution:

First draw the graphs of given functions.
Then find intersection points: $9 - x^2 = 3 - x \Longrightarrow x = 3$ and $x = -2$

Area= $\displaystyle\int_{-2}^{3} [9 - x^2 - (3 - x)]dx = \left(6x - \frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_{-2}^{3} = \frac{125}{6}$

5) Consider the matrices

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -4 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 7 \\ 0 & -2 \\ 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 6 & 0 & -3 \\ 4 & 8 & 2 \end{bmatrix}.$$

a) (4 pnt) Calculate DC if it is possible. If not explain why?

$$DC = \begin{bmatrix} 21 & 39 \\ 26 & 14 \end{bmatrix}$$

b) (4 pnt) Calculate C^T.

$$C^T = \begin{bmatrix} 5 & 0 & 3 \\ 7 & -2 & 1 \end{bmatrix}$$

c) (4 pnt) Calculate BC if it is possible. If not explain why?

$$BC = \begin{bmatrix} 18 & 30 \end{bmatrix}$$

d) (4 pnt) Calculate $AD - 3C^T$ if it is possible. If not explain why?

$$AD - 3C^T = \begin{bmatrix} -1 & 16 & 8 \\ -19 & 46 & 16 \end{bmatrix}$$

e) (4 pnt) Calculate $DA + 4C$ if it is possible. If not explain why?

It is not possible since the number of columns of D not equal to the number of rows of A.