ÇANKAYA UNIVERSITY

Department of Mathematics

MCS 102 - Mathematics for International Trade II

FINAL EXAMINATION

23.05.2017

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 100 minutes

Question	Grade	Out of
1		22
2		19
3		14
4		15
5		15
6		15
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) a) (5 pnts) If $A = \begin{bmatrix} 4 & -2 & 7 \\ 1 & -1 & 0 \\ 2 & 5 & 8 \end{bmatrix}$ find $\det(A)$.

$\det(A) = 33$

b) (6 pnts) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$ find A^{-1}.

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

a) (11 pnts) Solve the linear system (By using matrix reduction or Cramer's Rule.)

$$\begin{aligned} x + 2y - z &= 1 \\ 2x + 3y - z &= 0 \\ -3x - 4y + 2z &= 3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & -1 & 0 \\ -3 & -4 & 2 & 3 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \Longrightarrow x = -5,\ y = 4,\ z = 2$$

2) a) (11 pnts) If $f(x,y,z) = x^3y^2 - xz - e^{z^2} + \ln(x^2+z) + \ln y$, find f_x, f_y, f_z, f_{xx}, f_{xy}.

$$
\begin{aligned}
f_x &= 3x^2y^2 - z + \frac{2x}{x^2+z} \\
f_y &= 2x^3y + \frac{1}{y} \\
f_z &= -x - e^{z^2}2z + \frac{1}{x^2+z} \\
f_{xx} &= 6xy^2 + \frac{2z-2x^2}{(x^2+z)^2} \\
f_{xy} &= 6yx^2
\end{aligned}
$$

b) (8 pnts) If $f(x,y) = 3x^2y - 5xy + e^{x+y} - 5x$, find f_x, f_y, f_{yx}, f_{yy}.

$$
\begin{aligned}
f_x &= 6xy - 5y + e^{x+y} - 5 \\
f_y &= 3x^2 - 5x + e^{x+y} \\
f_{yx} &= 6x - 5 + e^{x+y} \\
f_{yy} &= e^{x+y}
\end{aligned}
$$

3) a) (8 pnts) By using chain rule find $\dfrac{\partial z}{\partial t}$ and $\dfrac{\partial z}{\partial r}$ for $z = 3x^2y^2 - y^3x$ if $x = r^2 + \ln t,\ y = r^3e^t$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (6xy^2 - y^3)\frac{1}{t} + (6x^2y - 3y^2x)r^3e^t$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = (6xy^2 - y^3)2r + (6x^2y - 3y^2x)3r^2e^t$$

b) (6 pnts) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $\quad xz^2 + \ln y = xyz - 5y.$

$F(x, y, z) = xz^2 + \ln y - xyz + 5y$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{z^2 - yz}{2zx - xy}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\frac{1}{y} - xz + 5}{2zx - xy}$$

4) Choose one of them.

(15 pnts) Examine $f(x, y) = x^2 - 2yx + \frac{y^3}{3} - 8y$ for relative extremum.

(20 pnts) Examine $f(x, y) = x^2y + yx - 2y - x^3 + 12x$ for relative extremum.

Solution:

$$
\begin{aligned}
(1) \quad f_x =& 2x - 2y = 0 \\
(2) \quad f_y =& -2x + y^2 - 8 = 0 \\
f_{xx} =& 2 \\
f_{yy} =& 2y \\
f_{xy} =& -2
\end{aligned}
$$

By (1) and (2) $y^2 - 2y - 8 = 0$ and $y = 4$, $y = -2$

By (1) $y = 4 \Rightarrow x = 4$ and $y = -2 \Rightarrow x = -2$. Then critical points are $P_1(4, 4)$, $P_2(-2, -2)$.

$D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = 4y - 4$

$D(P_1) = 12 > 0, \quad f_{xx}(P_1) = 2 > 0$ then P_1 is a relative minimum.

$D(P_2) = -12 < 0$ then P_2 is a saddle point.

5) (15 pnts) Find all critical points of the function $f(x,y) = x - 3y - 8$ subject to the constraint $x^2 + 3y^2 = 1$.

Solution:

Constraint $g(x,y) = x^2 + 3y^2 - 1$.

By Lagrange multipliers,

$$
\begin{array}{lll}
(1) & f_x = \lambda g_x & 1 = 2\lambda x \\
(2) & f_y = \lambda g_y & -3 = 6\lambda y \\
(3) & g(x,y) = 0 & x^2 + 3y^2 = 1
\end{array}
$$

By (1) $x = \dfrac{1}{2\lambda}$ and by (2) $y = \dfrac{-1}{2\lambda}$

By (3) $\dfrac{1}{4\lambda^2} + \dfrac{3}{4\lambda^2} = 1 \Longrightarrow \lambda = 1$ and $\lambda = -1$.

If $\lambda = 1$, $x = \dfrac{1}{2} y = \dfrac{-1}{2}$

If $\lambda = -1$, $x = \dfrac{-1}{2} y = \dfrac{1}{2}$

Then critical points are $P_1\left(\dfrac{1}{2}, \dfrac{-1}{2}\right)$ and $P_2\left(\dfrac{-1}{2}, \dfrac{1}{2}\right)$.

6) Evaluate.

a) (5 pnt) $\int \left(4x^3 - x^2 + \frac{1}{x^2} - 5\sqrt{x}\right) dx = x^4 - \frac{x^3}{3} - \frac{1}{x} - \frac{10}{3}x^{3/2} + C$

b) (5 pnt) $\int e^{x^2+1} x dx = \int e^u \frac{du}{2} = \frac{e^u}{2} + C = \frac{e^{x^2+1}}{2} + C$

$$u = x^2 + 1, \quad du = 2x dx$$

c) (5 pnt) $\int (x^3 - 3x)^7 (x^2 - 1)\, dx = \int u^7 \frac{du}{3} = \frac{u^8}{24} + C = \frac{(x^3 - 3x)^8}{24} + C$

$$u = x^3 - 3x, \quad du = (3x^2 - 3)dx$$